Robust Learning of Non-linear Mechanics using Latent Energy-based Neural Networks

Problem of interest

Large strain hyperelasticity framework
- Deformable bodies (beams, shells, volumes)
- Material nonlinearity (non-linear constitutive law)
- Geometric nonlinearity (large deformation)

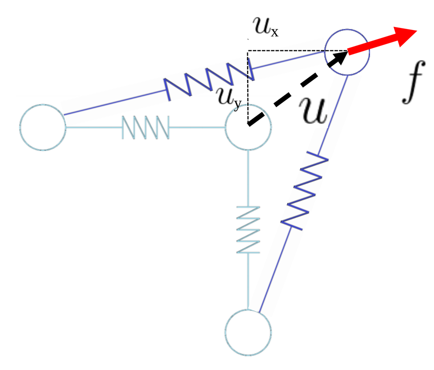
Let $u \in \mathbb{R}^n$ the discretized displacement field, $f \in \mathbb{R}^n$ the external forces and $\mathcal{U}$ the strain energy that depends only on u.

Under some assumptions, the following governing equation holds : $\frac{\partial \mathcal{U}}{\partial u} = f$

Displacement to Force (DtF) : finding the external force for a given displacement is direct, it is obviously unique

Force to Displacement (FtD) : finding a displacement field corresponding to a given external force require a root-finding algorithm. If $\mathcal{U}$ is nonconvex, the solution is not unique.

We aim to learn both DtF and FtD problems with a single model that should be reliable and have a low inference cost.



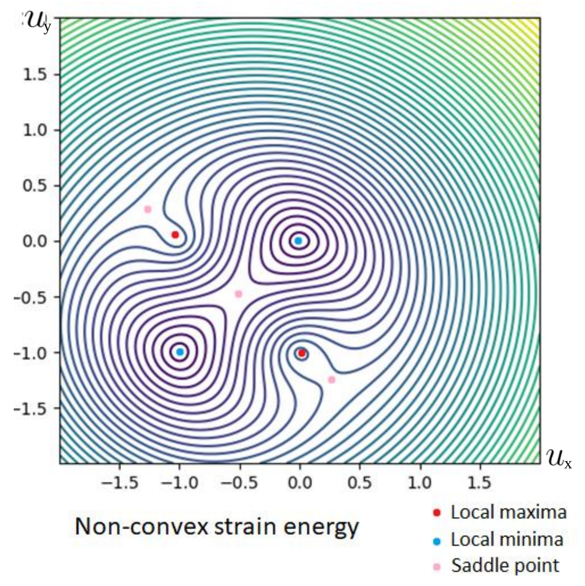


Fig 1. Example of simple system with non-convex strain energy

Proposed architecture and metodology

Latent Energy Based Neural Network (LEBNN)

Architecture :
- Based on an auto-encoder structure for both u and f
- The physics is learned in the latent space by learning to conserve a learnable latent energy ➔ Fig. 2

Training methodology :
- LEBNN is trained using a set of (u, f) equilibrium
- A DtF loss is used because the evaluation of the NN is direct
- Two anto-encoder loss terms are added to the training loss
- No supervision required for the latent energy $\widehat{U}$

Inference :
- DtF inference is direct
- FtD inference is made by using a gradient algorithm minimizing the deviation from equilibrium in latent space

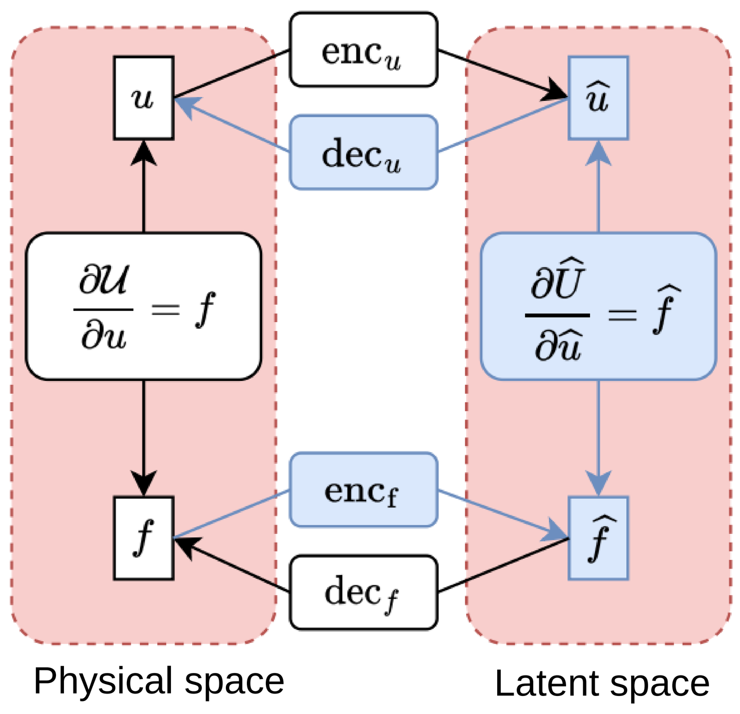


Fig 2. LEBNN Architecture

Experiments

Validation on several test cases

2 dofs test cases : ➔ Fig . 1
- Linearized two springs
- Nonlinear two springs on a convex subset
- Nonlinear two springs

Hyperelastic beam test case :
- 20 dof beam subject to a high force at its free end

Validation metrics :
- DtF MAE
- FtD MAE
- % of FtD inference leading to the right local minima

Latent physics :
- Direct FCNN FtD physics (DtF solved with Newton algorithm)
- Direct FCNN DtF physics (FtD solved with Newton algorithm)
- LEBNN physics

All architectures have approximately the same inference time.

	$\widehat{u} = \mathrm{NN}(\widehat{f})$	$\widehat{f} = \mathrm{NN}(\widehat{u})$	$\widehat{f} = \frac{\partial\,\mathrm{NN}(\widehat{u})}{\partial \widehat{u}}$
Linearized two springs			
- DtF MAE train	2.53 ± 0.30	3.31 ± 0.52	2.09 ± 0.17
- DtF MAE val	2.57 ± 0.30	3.34 ± 0.57	2.05 ± 0.22
- FtD MAE val	2.93 ± 0.26	2.52 ± 0.64	2.56 ± 0.25
- % succes val	100.00 ± 0.00	100.00 ± 0.00	100.00 ± 0.00
Two springs (convex subset)			
- DtF MAE train	6.25 ± 1.31	2.96 ± 0.39	2.37 ± 0.37
- DtF MAE val	6.10 ± 1.56	2.99 ± 0.47	2.38 ± 0.34
- FtD MAE val	3.24 ± 0.61	3.07 ± 0.90	3.92 ± 0.64
- % succes val	100.00 ± 0.00	100.00 ± 0.00	99.75 ± 0.25
Two springs			
- DtF MAE train	79.13 ± 36.54	4.35 ± 0.51	3.32 ± 0.36
- DtF MAE val	54.65 ± 24.45	4.13 ± 0.50	2.90 ± 0.48
- FtD MAE val	29.16 ± 2.11	11.67 ± 1.21	7.89 ± 1.89
- % succes val	88.00 ± 0.75	96.25 ± 0.75	97.50 ± 0.00
Hyperelastic beam			
- DtF MAE train	63.18 ± 10.51	7.35 ± 0.27	5.02 ± 0.55
- DtF MAE val	62.25 ± 13.85	7.94 ± 0.28	5.46 ± 0.54
- FtD MAE val	70.81 ± 0.21	17.85 ± 3.15	14.91 ± 1.19
- % succes val	86.90 ± 0.07	97.82 ± 0.73	97.88 ± 0.26

Fig 3. Comparison of 3 latent physics on several test cases

Results

For non-convex strain energy test cases, any structure for the latent physics works. For non-convex strain energy, the FCNN FtD physics obviously fail to learn the multiplicity of FtD solutions. ➔ Fig . 3

Both FCNN DtF and LEBNN learned correctly the non-convex test cases, but LEBNN is much more interpretable : the latent energy learned can be used to certify that the physics has been learned correctly and that no unphysics behaviors can occurs.

The learned energy has the same properties has the same convexity properties as the physical strain energy (same number of local maxima or minima, etc). ➔ Fig . 4

For the beam, LEBNN allow to visualize the latent energy in a 2 dimensional space instead of a 20 dimensional required to visualize de strain energy ➔ Fig . 5

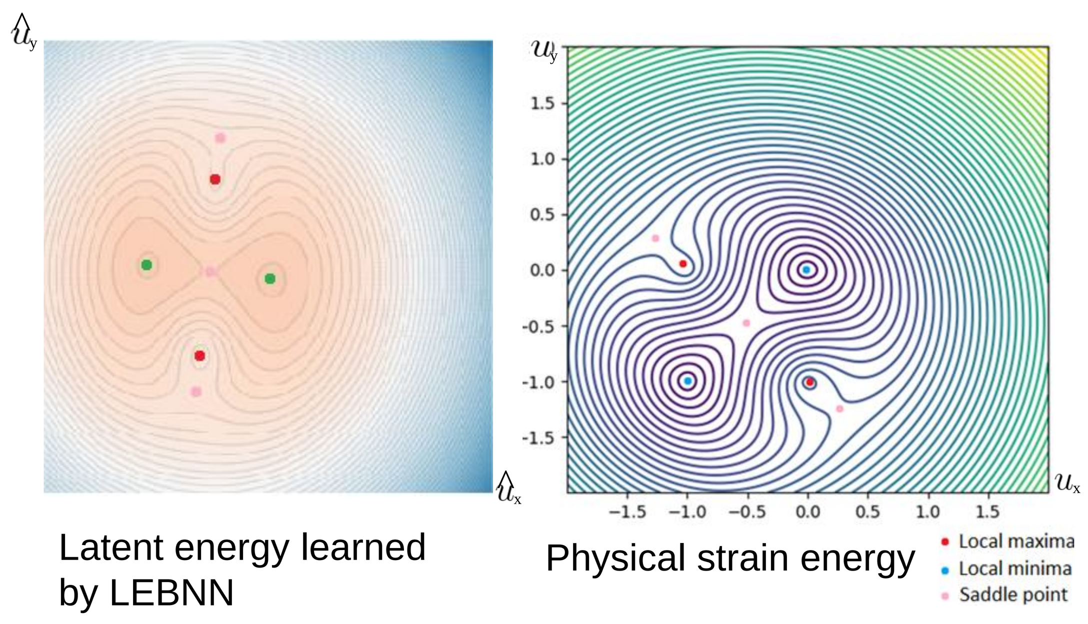


Fig 4. Comparison of learned energy and physical strain energy

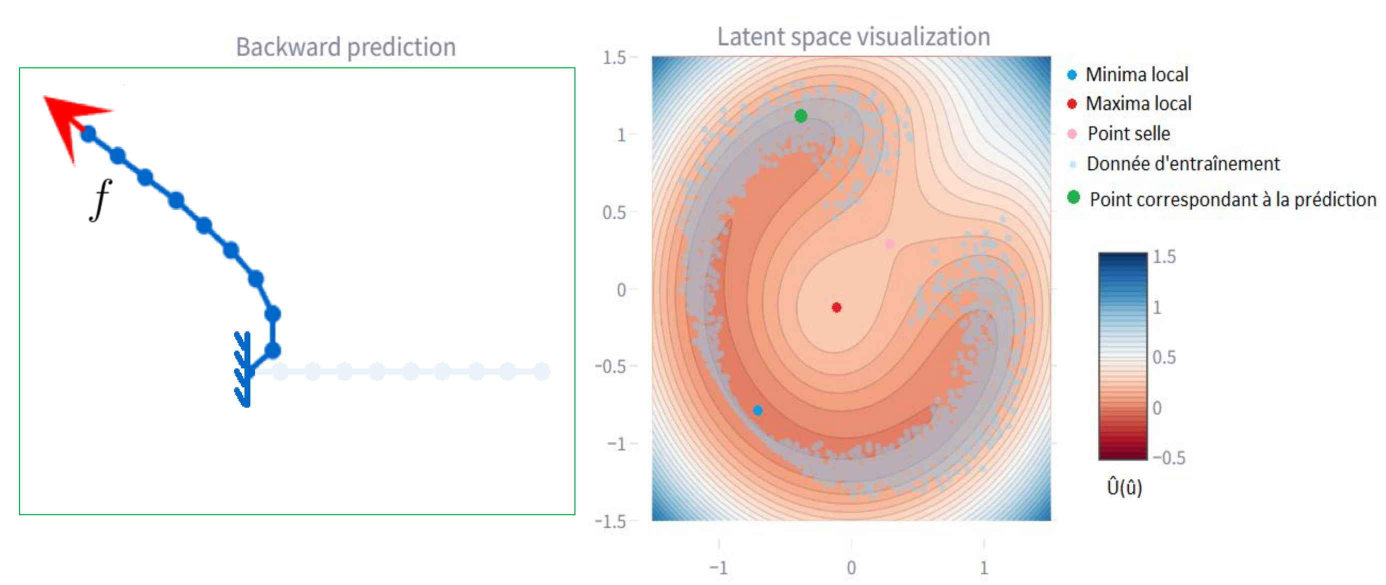


Fig 5. Predictred displacement (left) and latent energy (right) for the 20dof beam test case

Try by yourselves ! https://lebnnbeam-1qhu98q9jlfi.streamlit.app

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